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BAYESIAN INVERSION OF MICROTREMOR ARRAY DISPERSION DATA FOR V_S STRUCTURE ON THE FRASER RIVER DELTA, BRITISH COLUMBIA

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ABSTRACT

This paper develops a Bayesian inversion approach to estimate the subsurface shear-wave velocity profile from microtremor array dispersion data, which is applied to a site on the Fraser River delta, British Columbia. Microtremors were recorded using an array of five seismographs set in an expanding series of apertures between 5-10 m and 160-180 m that provided phase velocity values over the frequency band of 1.2 to 6.7 Hz. The shear-wave velocity model parameterization is assessed using the Bayesian information criterion, which indicates a power law depth relation. Excellent agreement is obtained between the inversion result (optimal V_S profile with 95% credibility interval) and the average V_S profile determined from invasive methods (down-hole and seismic cone penetration testing) to over 100 m depth.

Introduction

The Fraser River delta (Fig. 1) is located in southwestern British Columbia at the northern end of the Cascadia Subduction Zone. Here, moderate seismicity coupled with a large population and important infrastructure results in the highest seismic risk in Canada (Onur et al., 2005). Seismic hazard assessment methodologies applied worldwide utilise the average shear-wave velocity of the upper 30 m of the subsurface (i.e. V_{S30}). A variety of seismic techniques have been developed to characterize the subsurface shear-wave velocity profile to estimate V_{S30} for this purpose. Much effort is currently devoted to determine methods to provide the V_{S30} estimate quickly and economically. Non-invasive seismic methods that keep all equipment at the surface are inherently less expensive than invasive methods such as down-hole measurements or seismic cone penetration testing (SCPT). This paper considers estimating the V_S profile and its uncertainty from microtremor data.

Microtremors are short period vibrations that result from coastal effects, atmospheric loading, wind interaction with structures and vegetation, and cultural sources such as traffic, trains, construction, and factories. Passive seismic techniques that seek to determine subsurface properties based on continuously-available, wide-band (0.02-50 Hz) seismic noise have become increasingly popular worldwide because they require little equipment and are unobtrusive to the site, resulting in relatively fast, low-cost measurements. Such methods are particularly useful in urban areas, and complement invasive geotechnical and/or active geophysical methods. The microtremor array method is based on recording background seismic noise using a spatial array of several

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seismographs to extract the Rayleigh wave dispersion curve, which can then be inverted for the V_S profile of the site. In 2006, at the 3rd International Symposium on the Effects of Surface Geology on Seismic Motion, critical issues for future improvement of the microtremor array method were identified to be the introduction of prior information and quantitative and meaningful evaluation of confidence intervals on V_S profiles (Cornou et al., 2006).

In this paper, we present the V_S profile (i.e. the optimal model) at a site on the Fraser River delta in southwest British Columbia estimated using Bayesian inversion of the dispersion curve determined from microtremor array measurements. Bayesian inversion considers the model to be a random variable constrained by data and prior information, and seeks properties of the posterior probability density (PPD) that represent optimal parameter estimates and parameter uncertainties. Defining an appropriate model parameterization is an important issue in inverse problems, which we address here using the Bayesian information criterion (BIC). The subsurface is modelled as a stack of horizontal and homogeneous layers characterized by four parameters: V_S, compressional-wave velocity, density, and layer thickness, overlying a uniform velocity half-space. Parameterizations considered include layers with constant velocities, constant velocity gradients, and power law gradients. The recovered V_S profile (with uncertainties) is compared with existing V_S-depth measurements made by down-hole and SCPT invasive methods.

Figure 1. Location of the Fraser River delta. Microtremors were recorded at the position of the white circle (inset).

Data Collection and Processing

Microtremor array measurements were conducted on the Fraser River delta within 30 m of the Geological Survey of Canada borehole FD94-4 which penetrated 300 m into the subsurface (Dallimore et al., 1995). V_S profiles are also available from three SCPT sites located 350 m south of the borehole, and a fourth site 570 m to the south. Holocene deltaic sands and silts compose the upper 235 m of the borehole, overlying over-consolidated Pleistocene glacial material. From compilation of ~500 V_S measurements across the Fraser River delta, the gross shear-wave velocity-depth structure increases significantly in the upper 100 m as a result of loading, best represented by a power law gradient (Hunter and Christian, 2001). The Holocene-Pleistocene boundary is always characterized by an abrupt increase in V_S by a factor of 1.5 to 3.0 (Hunter et al., 1998).

Microtremor recordings were collected on the Fraser River delta in a cross-shaped array geometry using five broadband 3-component sensors. Recordings were made for six different array apertures with minimum and maximum limits of 5-10 m and 160-180 m, respectively. Frequency-wavenumber (*f-k*) techniques were applied to extract the dispersion curve of the fundamental mode of the Rayleigh wave from the vertical-component microtremor recordings. Phase velocities were calculated using both traditional *f-k* (Lacoss et al., 1969) and high resolution *f-k* analysis (Capon, 1969), as supplied in the "Sesarray" software package of Marc Wathelet (www.geopsy.org), and are combined into a single dataset as the results are indistinguishable from each other. Fig. 2 presents the dispersion curve for the Fraser River delta site which varies from ~ 400 m/s at 1.2 Hz to ~ 130 m/s at 6.7 Hz with 51 data at logarithmically spaced frequencies. The curve is segmented, dashed lines in Fig. 2, due in part to generating the full dispersion curve from different array apertures with non-overlapping reliable frequency ranges.

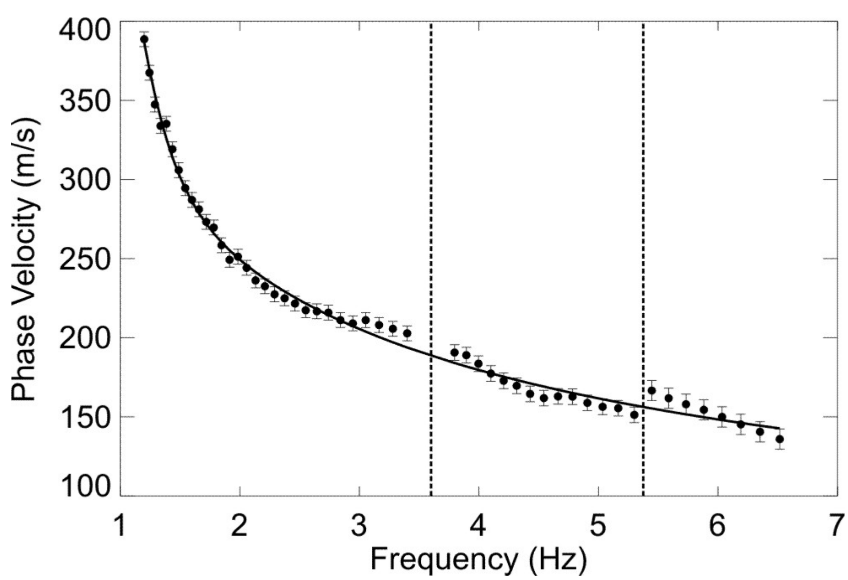

Figure 2. Dispersion curve for the Fraser River delta site. Maximum-likelihood standard deviation estimates indicated as error bars.

Inversion Methodology

A probabilistic formulation provides the full solution to the inverse problem in the form of the posterior probability density over the model space. Bayesian inversion is based on the assumption that the model represents a random variable which we seek to describe statistically. If **d** and **m** represent vectors of random data and model variables with N and M elements, respectively, Bayes' rule can be written

$$P(\mathbf{m} \mid \mathbf{d}) \propto P(\mathbf{d} \mid \mathbf{m}) P(\mathbf{m}), \tag{1}$$

where $P(\mathbf{m})$ represents prior information, and the conditional probability $P(\mathbf{d}|\mathbf{m})$ is interpreted as a function of **m** for the (fixed) measured data **d**, defining the likelihood function,

$$L(\mathbf{m}) \propto \exp[-E(\mathbf{m})], \tag{2}$$

where E is the data misfit function (considered below). Combining data and prior as a generalized misfit,

$$\varphi(\mathbf{m}) \equiv E(\mathbf{m}) - \log_e P(\mathbf{m}), \tag{3}$$

the PPD can be written

$$P(\mathbf{m} \mid \mathbf{d}) = \frac{\exp[-\varphi(\mathbf{m})]}{\int \exp[-\varphi(\mathbf{m}', \mathbf{d})] d\mathbf{m}'}, \tag{4}$$

where the domain of integration spans the M-dimensional parameter space.

The maximum *a posteriori* (MAP) or most probable model is estimated by maximizing the PPD:

$$\hat{\mathbf{m}} = \mathrm{Arg}_{\max}\{P(\mathbf{m} \mid \mathbf{d})\} = \mathrm{Arg}_{\min}\{\varphi(\mathbf{m})\}. \tag{5}$$

In this paper, $\varphi(\mathbf{m})$ is minimized numerically using adaptive simplex simulated annealing, a hybrid optimization algorithm that adaptively combines the local downhill simplex method within a very fast simulated annealing global search (Dosso et al., 2001). Model parameter uncertainties, such as the 95% highest-probability density (HPD) interval, are estimated from the PPD using the Markov-chain Monte Carlo method of Metropolis-Hastings sampling, applied for efficiency in a principal-component parameter space (Dosso et al., 2009).

Prior information considered in this paper consists of uniform distributions for each parameter on bounded intervals. Intervals are chosen to limit parameters to physically reasonable values, but are wide enough to allow the data (not the prior) to primarily determine the solution.

Determining an appropriate model parameterization is an important aspect of Bayesian inversion, which is addressed here by minimizing the BIC (Schwarz, 1978),

$$BIC = 2E(\hat{\mathbf{m}}) + M \log_e N , \tag{6}$$

over a number of possible parameterizations. Parameterizations considered differ in the number of layers and also in the representation of V_S over the layers. Shear-wave velocity models can be composed of combinations of three types of layers in which V_S is either constant, varies linearly with depth, or varies according to a power law relationship with depth.

Inversion Results

A variety of possible V_S model parameterizations were examined for inverting the Fraser River delta dispersion data. The preferred parameterization with the lowest BIC value is found for a model consisting of a power law layer over a uniform half-space. Fig. 2 shows the agreement between the dispersion curve computed for the MAP model (solid line) and the measured microtremor dispersion curve.

Specifying the data uncertainty (error) distribution, which defines the likelihood function, is an important practical aspect of Bayesian inversion. To estimate error statistics, independent identically-distributed (IID) Gaussian errors were initially assumed for a preliminary inversion. Statistical tests applied *a posteriori* to the standardized residuals indicated three frequency bands with differing error statistics (Fig. 2), with correlated errors over each band (these bands are consistent with the segments described earlier). An appropriate data covariance matrix was computed from the residuals and used in subsequent inversions.

Fig. 3 shows the recovered V_S profile consisting of the MAP model with a 95% HPD credibility interval. The shear-wave velocity of the power law layer is well constrained to 110 m depth. The invasive V_S measurements within the borehole and at four SCPT sites are averaged according to the logarithmic depth partitioning of the MAP model for which the mean value at each depth is plotted as a circle in Fig. 3 (error bars indicate one-standard deviation). Filled circles depict averaged down-hole and SCPT measurements to 60 m depth whereas open circles depict averaged down-hole only measurements. The mean V_S of these invasive methods closely approximates a power law depth relation and is in excellent agreement with the microtremor inversion result.

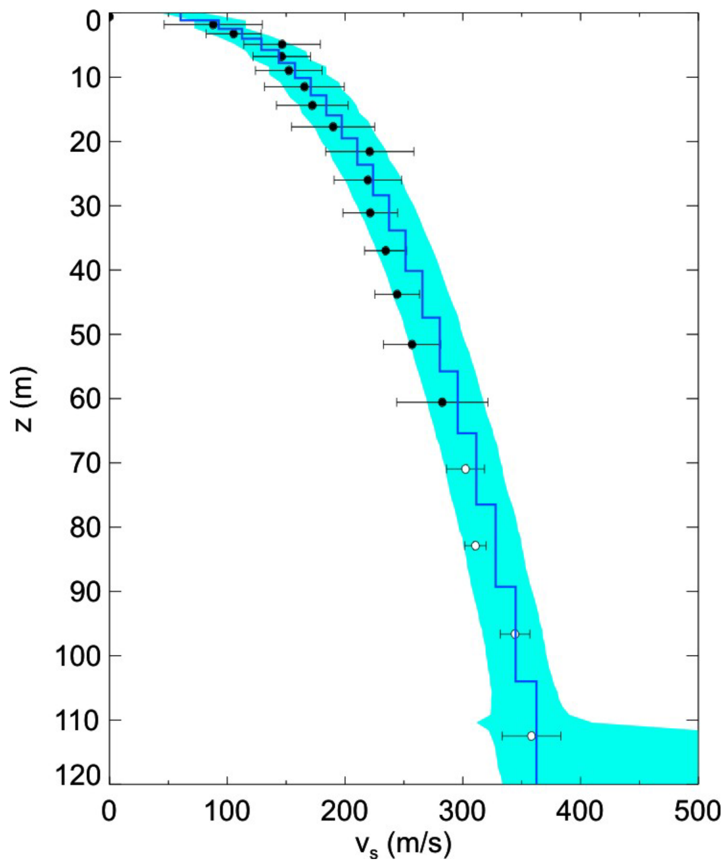

Figure 3. MAP model (solid line) and 95% HPD credibility interval (shaded region) from inversion at the delta site compared with V_S measurements from invasive down-hole and SCPT methods. Filled circles depict averaged down-hole and SCPT measurements to 60 m depth whereas open circles depict averaged down-hole only measurements.

Discussion and Conclusions

Microtremor inversion results are generally represented in terms of the minimum misfit solution, often without any uncertainty estimates. Several approaches exist to provide a relative measure of the inversion result's uncertainty, such as plotting all models visited during an optimization inversion that are within 10% of the minimum-misfit model (Parolai et al., 2007; Picozzi et al., 2009). This paper applies nonlinear Bayesian inversion of microtremor array dispersion data, with rigorous estimation of data error statistics and evaluation of an appropriate model parameterization, to determine the most-probable model of the subsurface V_S profile with quantitative uncertainty estimates. For the Fraser River delta site presented here, a well-resolved V_S profile to 110 m depth is determined from Bayesian inversion of phase velocities between frequencies of 1.2 to 6.7 Hz.

The type of model parameterization to use in shear-wave velocity modelling is usually unknown *a priori*. A common strategy to determine the parameterization is to progressively increase the number of layers (parameters) until the data misfit stops decreasing significantly (e.g. Renalier *et al.* 2009). However, this approach can add unnecessary profile structure. Instead, the Bayesian information criterion is used to provide an objective criterion for selecting the most appropriate model parameterization. The BIC indicates that the Fraser River delta dispersion data are best modelled using a power law depth relation. The mean V_S of the invasive methods closely approximates a power-law depth relation as previously noted for the upper 100 m of sediments of the Fraser River delta (Hunter and Christian, 2001).

All V_S-profiling methods considered here determine similar V_S values but penetrate to very different depths: SCPT, up to 60 m; microtremor array, 110 m; and down-hole, 300 m. All three methods are valid to use for, and produce very similar, V_{S30} estimates. The microtremor array method is an extremely promising V_S-profiling methodology for seismic hazard assessment due to its depth of penetration and consistency of V_S results with other invasive methods.

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